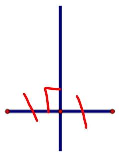
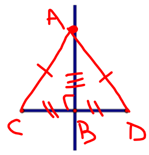
5.2 Use Perpendicular Bisectors

5.3 Use Angle Bisectors of Triangles

perpendicular bisector—is a line, segment, plane, or ray that passes through the midpoint of a segment and is perpendicular to it.



Thm 5.2—Perpendicular Bisector Theorem—In a plane, any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

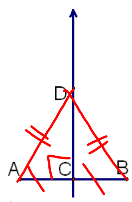


Given: $\overline{AB}$ perp. bis. of $\overline{CD}$
Prove: $\overline{AC} \cong \overline{AD}$

Why? Δs $\cong$ by SAS

Thm 5.3—Converse of the Perpendicular Bisector Theorem—In a plane, any point equidistant from the endpoints of the segment lies on the perpendicular bisector of the segment.

Examples:

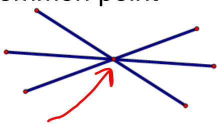


1. $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$.
   $m\angle DCA = 2x$. Solve for x. 45
   $90 = 2x$

   $AC = 3y + 2$, $BC = 14$. Solve for y. 4
   $3y + 2 = 14$

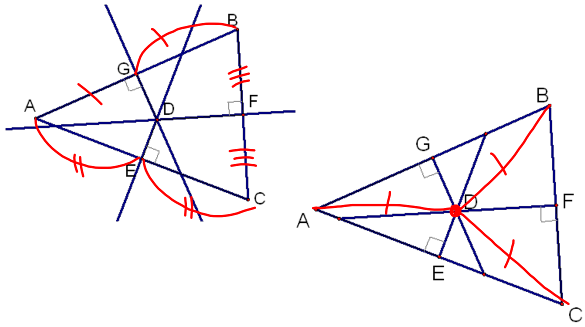
   $AD = 4z$, $BD = 20$. Solve for z. 5
   $4z = 20$
   $z = 5$

Concurrent lines—three or more lines that intersect at a common point



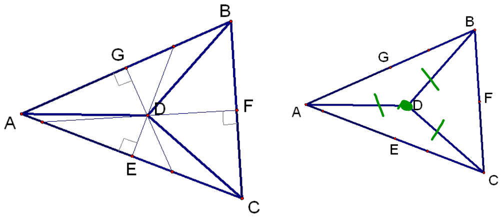
Point of concurrency—the point of intersection

Circumcenter—the point of concurrency of the perpendicular bisectors of a triangle.

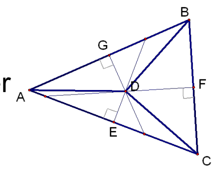


Thm 5.4 Circumcenter theorem—The circumcenter of a triangle is equidistant from the vertices of the triangle.

Conclusion: $\overline{AD} \cong \overline{CD} \cong \overline{BD}$



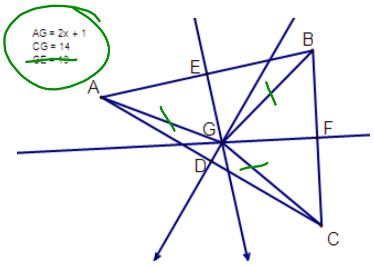
Proof:
Given: D is the circumcenter
Prove: $\overline{AD} \cong \overline{CD} \cong \overline{BD}$



Ex 2. G is the circumcenter. x = 6.5

$2x + 1 = 14$

AG = 2x + 1
CG = 14
~~GE = 10~~

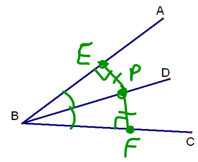


5.3 Angle Bisectors

Thm 5.5—Angle Bisector Theorem--Any point on the angle bisector is equidistant from the sides of the angle.

Thm 5.6—Converse of the Angle Bisector Theorem—If a point is on the interior of an angle and it is equidistant from the sides of the angle, then it lies on the angle bisector.

Conclusion: $\overline{PE} \cong \overline{PF}$



Ex 3. $\overline{BD}$ bisects ∠ABC. Solve for x. $\frac{16}{5}$

m∠ABC = 15x

m∠ABD = 10x – 8

$\frac{1}{2}15x = 10x - 8$

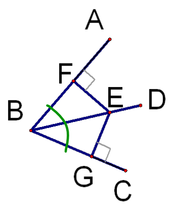
$15x = 2(10x - 8)$

EF = y + 8

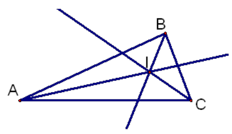
EG = 3y – 6

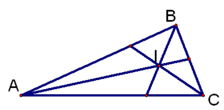
y = 7

$y + 8 = 3y - 6$



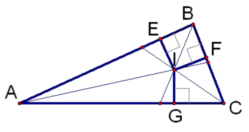
Incenter—The point of concurrency of the angle bisectors of a triangle.

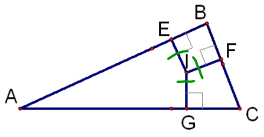




Thm 5.7 The Incenter Theorem—The incenter of a triangle is equidistant from each side of the triangle.

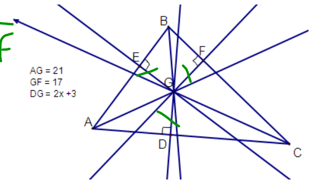
Conclusion: $\overline{EI} \cong \overline{IG} \cong \overline{IF}$





Ex 4. G is the incenter. x = 7.

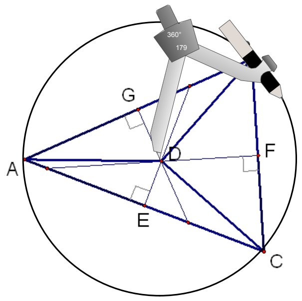
$\overline{GD} \cong \overline{GE} \cong \overline{GF}$

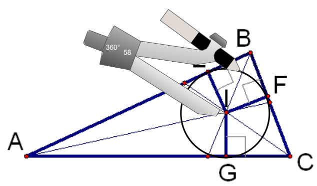


$2x+3=17$

$x=7$

Circles can be drawn around these centers.





HW
p307 #s 11-17
p309 quiz #s 1-3
p313-314 #s 3-8, 12-17,